NEW MODEL AND ANALYTICAL REVIEW OF APPROACHES TO BUCKLING PROBLEM INVESTIGATION OF STRUCTURALLY-ANISOTROPIC AIRCRAFT PANELS MADE FROM COMPOSITE MATERIALS

L M Gavva
PhD, Researcher of «Aircraft Design» Department, Acc. Professor of «Machine components» Department at Moscow Aviation Institute (NRU), Moscow, Russia
E-mail: rva101@mail.ru

Abstract The different approaches were analyzed to investigate the buckling problems of structurally-anisotropic panels made from composite materials. One considered the studies of scientific schools from 2000 year to present time, mainly. The classification of mathematical models, analytical methods of calculations, numerical methods of calculations, testing results is presented in this review.

Aircraft composite structure design in the field of production technology is the outlook research trend. New mathematical model relations for the buckling investigation of structurally-anisotropic panels comprising composite materials are presented in this study. The primary scientific novelty of this research is the further development of the theory of thin-walled elastic ribs related to the contact problem for the skin and the rib with an improved rib model. One considers the residual thermal stresses and the preliminary tension of the reinforcing fibers with respect to panel production technology.

The mathematical model relations for the pre-critical stress state investigation of structurally-anisotropic panels made of composite materials are presented. Furthermore, the mathematical model relations for the buckling problem investigation of structurally-anisotropic panels made of composite materials are presented in view of the pre-critical stress state. The critical force definition of the general bending mode of the thin-walled system buckling and the critical force definition of the multi-wave torsion buckling are of the most interest in accordance with traditional design practices. In both cases, bending is integral with the plane stress state. Thus, the buckling problem results in the boundary value problem when solving for the eighth order partial derivative equation in the rectangular field**.** The schematization of the panel as structurally-anisotropic has been proposed as a design model when and the critical forces of total bending mode of buckling are determined. For a multi-wave torsion buckling study, one should use the generalized function set. The solution is designed by a double trigonometric series and by a unitary trigonometric series. A computer program package is developed using the MATLAB operating environment. The computer program package has been utilized for multi-criteria optimization of the design of structurally-anisotropic aircraft composite panels. The influence of the structure parameters on the level of critical buckling forces for bending and for torsion modes has been analyzed. The results of testing series are presented. The results of new calculations are presented.

Keywords: review, panels made of composite materials, eccentric longitudinal and lateral set, thin-walled rib, non-symmetric package structure, force and technology temperature action, pre-critical stress state, buckling, bending mode, torsion mode.

1. Introduction

A new design problem – the design to cost – is possible to be solved in combining the high-precision models and modern computer technologies and decreasing the test amount. A computer program package in MATLAB was performed for the investigation and multi-criteria optimization of the buckling of structurally-anisotropic composite panels of FA.

The buckling problems of a flat rectangular multilayer panel made from polymer fiber composite materials with the eccentric longitudinal and lateral stiffening set are considered. The buckling problems of a flat rectangular composite panel being anisotropic due to non-symmetric package structure over the thickness are also discussed. The panels are subjected to the distributed constant compressive loading applied to the edges in the casing plane in the stationary temperature field. The boundary conditions at the contour are assumed to be the particular case with conformable boundary restrictions for the plane problem and problem of bending, the solution in closed form is designed by a trigonometric series.

One should take into consideration the technological factors occurring in the fabrication of composites, namely, residual thermal stresses arising during cooling after hardening and pre-stressed tension of reinforcing fibers that is performed in order to increase the bearing strength of the structure.

The refined statement of the buckling problems has been formulated subject to the pre-critical stress state in the compression of flat rectangular multiplied panels made of polymer fiber composite materials, the casing of which is eccentrically supported by the longitudinal-lateral stiffening set.

The buckling problem investigation of flat rectangular structurally-anisotropic composite panels is relevant for the design of the bearing surfaces of FA. The majority of the authors of the theoretical studies during the latest twenty years are paying due attention to the features of the composite panel deformation behavior. The classification and survey of the main directions of the buckling theory development of the structurally-anisotropic composite panels, the proposed mathematical models and equations are of the most interest.

Buckling Problem Statement:

[Setoodeh A.R., Karami G., 2003], [Gangadhara P.B., 2008], [Mittelstedt C., Schroder K.U., 2010], [Yshii L.N., Lucena N.E., Monteiro F.A.S., Santana R.C., 2018], [Ragb O., Matbuly M.S., 2017], [Castro S.G.P., Donadon M.V., 2017].

Buckling Problem Statement with Thermal and Force Loading:

[Shukla K.K., Nath Y., 2002], [Chen C.S., Lin C.Y., Chen R.D., 2011], [Matsunaga H., 2005], [Cetkovic M., 2016], [Cetkovic M., Gyorgy L., 2016], [Kettaf F.Z., Benguediab M., Tounsi A., 2015], [Naik N.S., Sayyad A.S., 2019].

Buckling Problem Statement subject to Production Technology:

[Chen X., Dai S.L., Xu K, 2001].

Analytical Methods to solve Buckling Problems:

[Pandey R., Shukla K.K., Jain A., 2009], [Vescovini R., Dozio L., 2015], [Kazemi M, 2015], [Yeter E., Erklig A., Bulut M., 2014], [Abramovich H., Weller T., Bisagni C., 2008].

Numerical Methods to solve Buckling Problems:

[Huang L., Sheikh A.H., Ng C.T., Griffith M.C., 2015], [Guo M.W., Harik I.E., Ren W.X., 2002], [Thankam V.S., Singh G., Rao G.V., Rath A.K., 2003], [Tenek L.T., 2001], [Tran l.V., Wahab M.A., Kim S.E., 2017], [Kumar S., Kumar R., Mandal S., Ranjan A., 2018], [Kumar S., Kumar R., Mandal S., Rahul A.K., 2018], [Zarei A., Khosravifard A., 2019], [Castro S.G.P., Donadon M.V., Guimaraes T.A.M., 2019].

Test Investigations:

[Falzon B.G., Stevens K.A., Davies G.O., 2000], [Park O., Haftka R.T., Sankar B.V., Starnes J.H., Nagendra S., 2001], [Rouse M., Assadi M., 2001], [Ungbyfkorn V., Singhatanadgid P., 2003], [Baker D.J., 2000], [Zhao W., Xie Z., Wang H., Li X., Hao J., 2019], [Bai R., Bao S., Lei

Z., Liu D., Yan C., 2018], [Kumar S., Kumar R., Mandal S., 2018], [Sanches M.L., De Almeida S.F.M., Carrillo J., 2017].

The critical force calculations for the general bending mode of the thin-walled system buckling and the critical force calculations for the multi-wave torsion buckling are of the most actual interest in accordance with traditional design practices. General bending buckling is characterized by the less number of semi-waves than the number of stringers. Multi-wave torsion buckling is characterized by skin bending between cross nodes and rib rotation without deformation of the profile cross section. In both cases, bending is integral with the plane stress state. The schematization of the panel as structurally-anisotropic has been proposed as a design model when the stress-strain state and critical forces of total bending mode of buckling are determined. For a multi-wave torsion buckling study, one should use the generalized function techniques.

New mathematical model relations for the pre-critical stress-strain state and for the buckling problem investigation of structurally-anisotropic panels comprising composite materials are presented in this study. The mathematical model of a stiffening rib being torsioned under one-sided contact with the skin is refined. The scientific novelty of this research reflects the further development of the theory of thin-walled elastic ribs related to the contact problem for the skin and rib with an improved rib torsion model. The aim of this study is the buckling problem statement and the approach to solve this problem in view of the non-uniform pre-critical stress state and production technology. The buckling problem results in the boundary value problem when solving for the eighth order partial derivative equation in the rectangular field**.** The buckling problem statement and the proposed approach to solve this problem are new and are of the interest from the design and manufacture of aircraft outlook specimen made from modern composite materials.

2. Buckling of structurally-anisotropic composite panels. Problem Statement

Both cases, the buckling problem as the problem of the pre-critical stress state are integral, they cannot be divided into a plane part and a plate bending.

The eighth order differential equation is resolved for the buckling problems (e.g. Firsanov and Gavva 2017). This equation is designed as an equilibrium differential equation with the effect of the reduced loading connected with the normal forces N_x, N_y and tangential forces N_{xy}, N_{yx}

$$\sum_{i=0,1,2...}^{8} K_{8\text{-}i,i} \frac{\partial^8 \Phi}{\partial x^{8-i} \partial y^i} = N_x \frac{\partial^2 w}{\partial x^2} + \left(N_{xy} + N_{yx} \right) \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \tag{1}$$

The deflection function $w(x,y)$ is coupled with the potential function $\Phi(x,y)$ as

$$w = \left(R_{40} \frac{\partial^4}{\partial x^4} + R_{31} \frac{\partial^4}{\partial x^3 \partial y} + R_{22} \frac{\partial^4}{\partial x^2 \partial y^2} + R_{13} \frac{\partial^4}{\partial x \partial y^3} + R_{04} \frac{\partial^4}{\partial y^4} \right) \Phi \tag{2}$$

The coefficients R_{ij} , $i=4,3,...,0$, $j=0,1,...,4$ in the relation formulas (2) and the coefficients K_{ij} , $i=8,7,...,0$, $j=0,1,...,8$ in the resolving equation (1) are the constant values, which depend on the elastic characteristics of the material and geometrical structure parameters.

For the composite panel of orthotropic structure the left-hand part of the eighth order differential equation is limited by the even-numbered partial derivatives of $\Phi(x,y)$ but the odd-numbered partial derivatives in the right-hand part are connected with the shear

$$\frac{K_{80}}{a^8}\frac{\partial^8\Phi}{\partial x^8}+\frac{K_{62}}{a^6b^2}\frac{\partial^8\Phi}{\partial x^6\partial y^2}+\frac{K_{44}}{a^4b^4}\frac{\partial^8\Phi}{\partial x^4\partial y^4}+\frac{K_{26}}{a^2b^6}\frac{\partial^8\Phi}{\partial x^2\partial y^6}+\frac{K_{08}}{b^8}\frac{\partial^8\Phi}{\partial y^8}=$$

$$=\begin{bmatrix} \frac{N_xR_{40}}{a^6}\frac{\partial^6\Phi}{\partial x^6} & +\frac{\left(N_{xy}+N_{yx}\right)R_{40}}{a^5b}\frac{\partial^6\Phi}{\partial x^5\partial y}+ \\ +\frac{\left(N_xR_{22}+N_yR_{40}\right)}{a^4b^2}\frac{\partial^6\Phi}{\partial x^4\partial y^2} & +\frac{\left(N_{xy}+N_{yx}\right)R_{22}}{a^3b^3}\frac{\partial^6\Phi}{\partial x^3\partial y^3}+ \\ +\frac{\left(N_xR_{04}+N_yR_{22}\right)}{a^2b^4}\frac{\partial^6\Phi}{\partial x^2\partial y^4} & +\frac{\left(N_{xy}+N_{yx}\right)R_{04}}{ab^5}\frac{\partial^6\Phi}{\partial x\partial y^5}+ \\ +\frac{N_yR_{04}}{b^6}\frac{\partial^6\Phi}{\partial y^6} & \end{bmatrix}, \qquad (3)$$

$x = x/a$, $y = y/b$ are the dimensionless coordinates related to the panel length a and to its width b.

All components of the stress-strain state including the inner force factors are related with the potential function $\Phi(x, y)$ as

$$\left.\begin{aligned} N_x &= L_{Nx}\Phi - N_x^T - N_x^H, \quad N_y = L_{Ny}\Phi - N_y^T - N_y^H \\ N_{xy} &= L_{Nxy}\Phi - N_{xy}^T - N_{xy}^H, \quad N_{yx} = L_{Nyx}\Phi - N_{yx}^T - N_{yx}^H \end{aligned}\right\} \qquad (4)$$

The linear differential operator for the orthotropic structure, for example

$$L_{Nx} = P_{60}^x\frac{\partial^6}{\partial x^6}+P_{42}^x\frac{\partial^6}{\partial x^4\partial y^2}+P_{24}^x\frac{\partial^6}{\partial x^2\partial y^4}+P_{06}^x\frac{\partial^6}{\partial y^6},$$

The thermal forces and moments are $N_x^T, N_y^T, N_{xy}^T, N_{yx}^T$, the tension forces and moments of the composite fibers are $N_x^H, N_y^H, N_{xy}^H, N_{yx}^H$. The coefficients P_{ij}^x, $i=6,4,2,0$, $j=0,2,4,6$ as the coefficients in the relation formulas (2), depend on the elastic characteristics of the material and geometrical structure parameters.

The buckling problem for a structurally-anisotropic composite panel is a nonlinear one according to equation (3) and connection formulas (4). The linearization method is used to determine the critical forces for the approximate solution. First, one considers the stress-strain state of the structure at compression, namely, the pre-critical main stress state being complicated as it is not divided into the plane problem and bending problem according to the mathematical model proposed. It is necessary to determine the distribution law of the normal and shear inner forces caused by the external loading. Then one considers the buckling problem as the proper value problem to obtain the additional displacement of the basis surface.

3. Pre-critical stress-strain state of structurally-anisotropic composite panels in compression

We consider the pre-buckling stress state of a flat rectangular composite panel with the eccentric stiffening set being orthotropic one. The panel is subjected to the uniform distributed normal compressive loading with P intensity applied to the lateral opposite sides in the skin plane. Boundary restrictions satisfy the hinging condition in respect to bending and the sliding constraint condition in the tangential direction for the plane problem when the panel is loaded by the shear force flows along its longitudinal edges.

First, it is necessary to determine the distribution of the normal forces N_x caused by the external loading with the coordinates x and y. The compressive loading is applied at two opposite sides in the longitudinal direction. It is of interest to compare the critical parameter results with the uniform pre-critical stress state and with the variable pre-critical stress state. Then one does not consider the distribution of inner forces N_y, N_{xy}, N_{yx}.

The equation (3) is uniform

$$\frac{K_{80}}{a^8}\frac{\partial^8\Phi}{\partial x^8}+\frac{K_{62}}{a^6b^2}\frac{\partial^8\Phi}{\partial x^6\partial y^2}+\frac{K_{44}}{a^4b^4}\frac{\partial^8\Phi}{\partial x^4\partial y^4}+\frac{K_{26}}{a^2b^6}\frac{\partial^8\Phi}{\partial x^2\partial y^6}+\frac{K_{08}}{b^8}\frac{\partial^8\Phi}{\partial y^8}=0. \quad (5)$$

The boundary restrictions are

$$\left.\begin{array}{ll} x=\pm 1 & w=M_x=v_0=0,\quad N_x=-P \\ y=0\ y=1 & w=M_y=u_0=N_y=0 \end{array}\right\} \quad (6)$$

The integral of the equation (5) that satisfies the edge conditions (6) is presented by the single trigonometric series. The boundary value problem is symmetric with respect to the coordinate x, the solution contains only even-numbered functions and the generalized displacement function $\Phi(x,y)$ is determined by four unknown constants

$$\Phi(x,y)=\sum_{i=1,3,5}^{\infty}\left[\sum_{L=1}^{4}B_{iL}ch(\lambda_{iL}x)\right]\sin(i\pi y) \quad (7)$$

Here $\lambda_{iL}=z_L\lambda_{iy}a$, $\lambda_{iy}=\frac{i\pi}{b}$, z_L are the roots of the corresponding characteristic polynomial and are calculated using the MATLAB operating environment.

The vector displacement components, the strains and the curves of the basis surface, the rotation angels and the inner force factors are derived by single trigonometric series accurate within the constants B_{iL} (7). The thermal and tension forces and moments are also derived by single trigonometric series. The symmetric SSS components contain only even-numbered functions with respect to the coordinate x, the oblique symmetric components consist of odd-numbered functions. The linear differential operators of the relations between the SSS kinematic and static components and the resolving potential function $\Phi(x,y)$ are known.

It is necessary to satisfy the face boundary conditions of the structure with $x = +1$ to determine the unknown constants B_{iL} ($i = 1, 3, 5,\ldots,\ L = 1, 2, 3, 4,$) and SSS components.

The normal forces N_x, corresponding to the pre-critical stress state of the stiffened composite panel compressed along the x axis are distributed as

$$N_x=P\sum_{i=1,3,5\ldots}^{\infty}\left\{\left[\sum_{L=1}^{4}(N_x)_{iL}\,ch(\lambda_{iL}x)\right]-\frac{(N_x^T)_i}{P}-\frac{(N_x^H)_i}{P}\right\}\sin(i\pi y) \quad (8)$$

$(N_x)_{iL}$ are the coefficients of single trigonometric series for the normal forces N_x, known after the determination of the constants B_{iL}, $(N_x^T)_i$, $(N_x^H)_i$ are the coefficients of single trigonometric series for the thermal and tension forces.

The variations of the transverse forces N_y and shear forces N_{xy}, N_{yx} are obtained analog to (8).

One can estimate the influence of production technology factors on the bearing strength of structurally-anisotropic composite panels if the non-uniform pre-critical stressed state is considered**,** boundary conditions are non-conformable as (6), and the solution is formed by a unitary trigonometric series.

4. Pre-critical stress-strain state of structurally-anisotropic composite panels in compression. Numerical results

A computer software package was developed in the MATLAB operating environment according to the algorithm presented. Computer programs are intended for the pre-critical stress state analysis and the computer multi-criteria optimization of the design for structurally-anisotropic aircraft composite panels.

As an example, Figure 1 presents the epure of the normal forces N_x of a flat rectangular panels made from carbon plastic eccentrically stiffened in the longitudinal direction. The panels are under the action of uniform compressive loading per unit length.

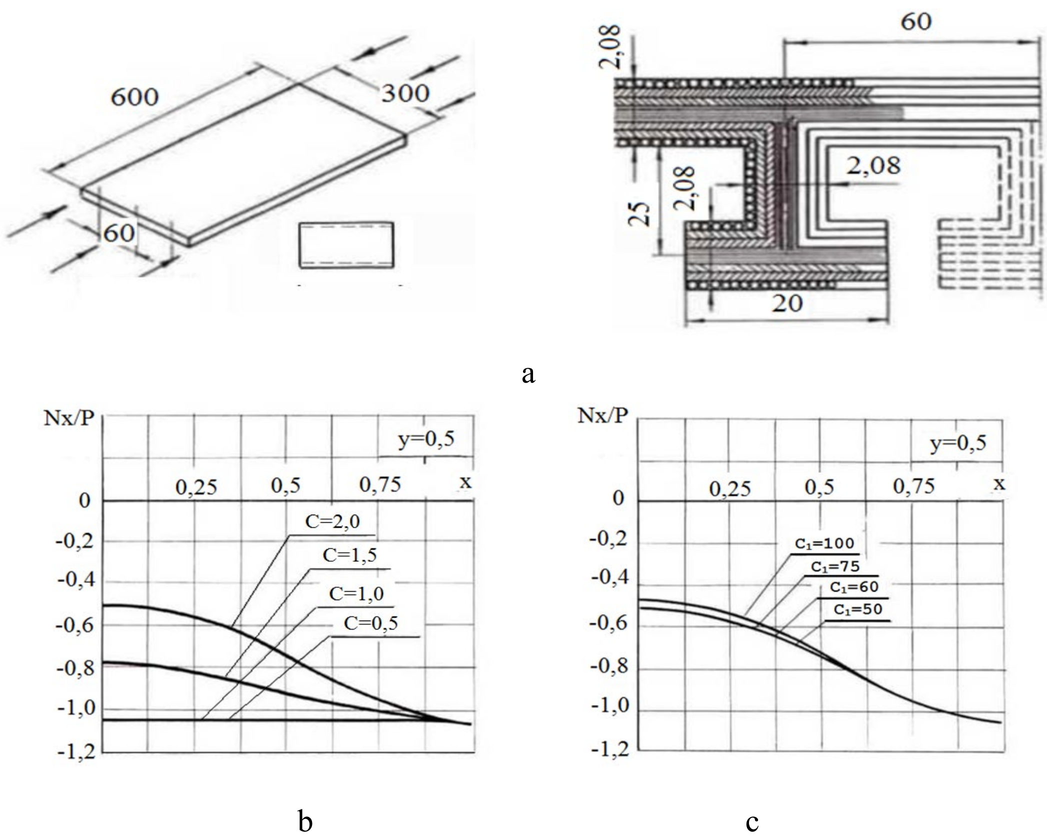


Figure 1. Stringer panel compressed in the longitudinal direction: (a) panel sizes, layout of the package and stringer geometry; (b) dependence of normal forces on panel side ratio under in-plane bending; (c) dependence of normal forces on stringer distance under in-plane bending

The shorter skin panel and the higher stringer stiffness, the pre-critical stress state is closer to uniform. If the panel side ratio $c < 1{,}0$, the inner normal forces are distributed along the panel length almost uniformly.

If the panel side ratio $c = 2{,}0$, the distribution of the normal forces per panel length under in-plane bending does not practically depend on the stringer distance.

5. Buckling of structurally-anisotropic composite panels subject to pre-critical stress state

The linear buckling problem is formulated as the proper value problem.

The general differential equation of the curved surface (3) determines the additional equilibrium state of the structure in view of the initial pre-buckling state. With the relation formulas (2) and normal force distribution (8) it is written as

$$\frac{K_{80}}{a^8}\frac{\partial^8\Phi}{\partial x^8}+\frac{K_{62}}{a^6b^2}\frac{\partial^8\Phi}{\partial x^6\partial y^2}+\frac{K_{44}}{a^4b^4}\frac{\partial^8\Phi}{\partial x^4\partial y^4}+\frac{K_{26}}{a^2b^6}\frac{\partial^8\Phi}{\partial x^2\partial y^6}+\frac{K_{08}}{b^8}\frac{\partial^8\Phi}{\partial y^8}= \tag{9}$$

$$=P\left\{\sum_{i=1,3,5...}^{\infty}\left[\left[\sum_{L=1}^{4}(N_x)_{iL}\,ch(\lambda_{iL}x)\right]-\frac{(N_x^T)_i}{P}-\frac{(N_x^H)_i}{P}\right]\sin(i\pi y)\right\}\left[\frac{R_{40}}{a^6}\frac{\partial^6\Phi}{\partial x^6}+\frac{R_{22}}{a^4b^2}\frac{\partial^6\Phi}{\partial x^4\partial y^2}+\frac{R_{04}}{a^2b^4}\frac{\partial^6\Phi}{\partial x^2\partial y^4}\right]$$

The forces N_x (8) are the variable coefficients in the equation (9).

The integral of the equation (9) satisfying to the uniform boundary conditions

$$\left.\begin{array}{ll} x=\pm 1 & v_0=N_x=w=M_x=0 \\ y=0\ \ y=1 & u_0=N_y=w=M_y=0 \end{array}\right\}, \tag{10}$$

may be approximated by a double trigonometric series. But it is not feasible to solve the problem in close form as the method is reduced with an infinite system of the linear algebraic equations.

The one-member approximation is considered as the first solution

$$\Phi(x,y)=f_{mn}\sin\left[\frac{m\pi}{2}(x+1)\right]\sin(n\pi y), \tag{11}$$

m and n are the wave parameters.

The critical force formula is designed with the orthogonalization procedure of the equation (9) subject to the solution (11). The expression

$$P=P^*\left\{\frac{\pi}{\sum\limits_{i=1,3,5...}^{\infty}\left(-\frac{2}{i}-\frac{1}{2n-i}+\frac{1}{2n+i}\right)\left\{\left[\sum\limits_{L=1}^{4}(N_x)_{iL}\frac{\left[(m\pi)^2-\lambda_{iL}^2\right]}{\left[(m\pi)^2+\lambda_{iL}^2\right]}\frac{sh\lambda_{iL}}{\lambda_{iL}}\right]-\frac{(N_x^T)_i}{P}-\frac{(N_x^H)_i}{P}\right\}}\right\} \tag{12}$$

provides the range of values P for additional possible deformation of the base surface at m = 1, 2, 3,…, n = 1, 2, 3,…

The P with «*» is the critical force calculated with the main uniform pre-buckling stress state (e.g. Firsanov and Gavva 2017)

$$P^*=\frac{\pi^2}{b^2}\frac{K_{80}\left(\frac{m}{c}\right)^8+K_{62}\left(\frac{m}{c}\right)^6n^2+K_{44}\left(\frac{m}{c}\right)^4n^4+K_{26}\left(\frac{m}{c}\right)^2n^6+K_{08}n^8}{\left[R_{40}\left(\frac{m}{c}\right)^4+R_{22}\left(\frac{m}{c}\right)^2n^2+R_{04}n^4\right]\left(\frac{m}{c}\right)^2},$$

for the general bending mode of buckling, the panel side ratio c = $2a/b$, a, b are the panel half-length and width, correspondingly.

The formula for the critical loading P of the multi-wave torsion buckling problem coincides to the formula (12) within the coefficients K_{ij} and R_{ij}. The coefficients $\hat{K}_{ij}$, $i,j = 0, 2, 4, 6, 8$ and $\hat{R}_{ij}$, $i,j = 0, 2, 4$ are determined by generalized stiffness characteristics while stiffness averaging for the elements of longitudinal set up to the skin is replaced with the discrete characteristics

$$\frac{1}{c_1} \to \begin{cases} \dfrac{2}{b}\sum_{i=1}^{N}\sin^2\left(n\pi y_i\right) \\ \dfrac{2}{b}\sum_{i=1}^{N}\cos^2\left(n\pi y_i\right) \end{cases} \to \hat{K}_{ij}, \hat{R}_{ij} \quad ,$$

c_1 is the stringer distance; y_i is the coordinate y of the discrete stringer.

It is allowable to estimate the influence of production technology factors on the bearing strength of the structurally-anisotropic composite panels if the complicated pre-critical stress state is considered and the solution is derived in a single approximation of the trigonometric series (11)**.** One considers the residual thermal stresses arising during cooling after hardening and the pre-stressed tension of the reinforcing fibers with respect to panel production technology.

The step by step method is used to determine the critical forces. The critical force P with «*» calculated with the uniform pre-buckling stress state is proposed as an initial first approach.

6. Buckling of structurally-anisotropic composite panels subject to pre-critical stress state. Results and Discussion

A computer program package is developed using the MATLAB operating environment. The computer program package has been utilized for the determination of the critical forces subject to the uniform and non-uniform pre-buckling stress state and for multi-criteria optimization of the design of structurally-anisotropic aircraft composite panels.

The results of determining the critical parameters for rectangular carbon plastic panels eccentrically stiffened and compressed in the longitudinal direction (Figure 1a) are presented in Figure 2 as an example in view of the uniform pre-buckling stress state.

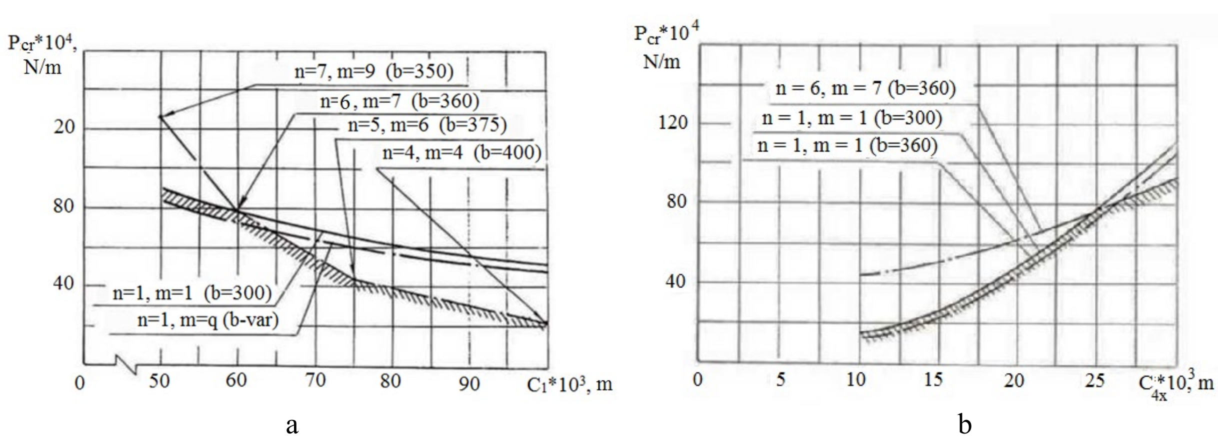


Figure 2. Stringer panel compressed in the longitudinal direction: dependence of critical forces of bending mode and torsion mode of buckling (a) on stringer distance; (b) on height of stringer cross section wall

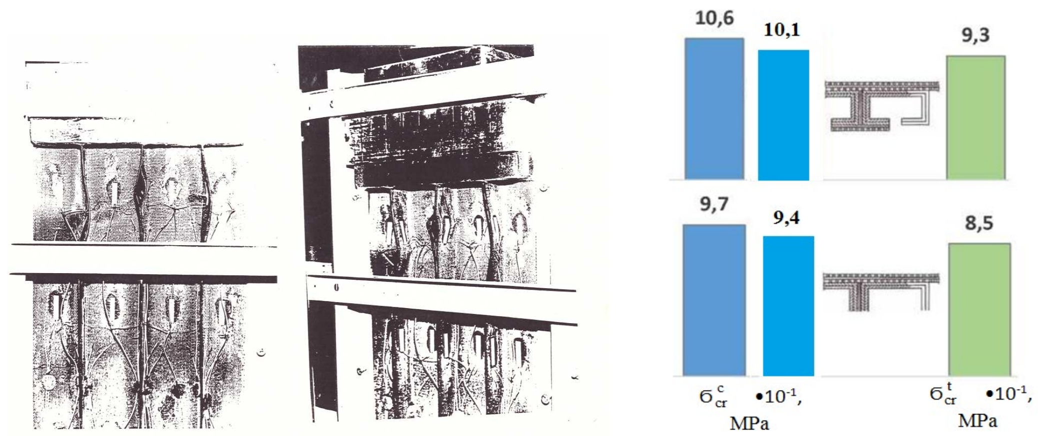
(———— *structurally-anisotropic model*; – · – · – · *discrete ribs*)

For a short panel, if $c < 2{,}0$, the multi-wave torsion buckling occurs: $n = 6$, $m = 2, 4, 5, 7, 11$. If $c = 2{,}0$, the panel is equally buckled. The buckling for panels with $c > 2{,}0$ has the general bending mode $n = 1$, $m = 1$.

The shorter stringer distance $c_1 < 60$ mm, the higher is probability for the general bending buckling mode $n = 1$, $m = 1$. If $c_1 = 60$ mm, the panel is equally buckled. While the stringer distance $c_1 > 60$ mm, the panel is characterized by torsion buckling mode $n = 4, 5, 6$, $m = 4, 6, 7$.

The lower bending stiffness of the stringer, if the height of stringer cross section wall $c_{4x} < 25$ mm, the higher is probability for the general bending buckling mode $n = 1$, $m = 1$. If $c_{4x} = 25$ mm, the panel is equally buckled. While the height of stringer cross section wall $c_{4x} > 25$ mm, the panel is characterized by torsion buckling mode $n = 6$, $m = 7$.

The testing series of uniform compressed stiffened composite panels for carrying the objects to the moment of stability loss (Figure 3) have been made using the special fixture for the proposed mathematical model verification.



a b

Figure 3. Stringer panels compressed in the longitudinal direction: (a) experimental specimen; (b) comparison of the test and calculation results for critical stresses

The refined theoretical results and the experimental data are in agreement qualitatively with respect to buckling modes and quantitatively with respect to critical stresses within 12 - 13% if pre-buckling stress state is considered uniform. And they reveal a higher precision of 8 - 10% in view of non-uniform main pre-buckling stress state. Thus, it confirms the authenticity of the presented mathematical model.

7. Conclusion

Since the solution is obtained by analytical methods, the calculation time is minimal. This is of interest from the perspective of practical design using parametric analysis. The results of the stress analysis calculations, as well as the results of the buckling analysis calculations, offer opportunities for reducing and optimizing the weight characteristics of aircraft elements.

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