Spacecraft Attitude Maneuver Using Fast Terminal Sliding Mode Control Based on Variable Exponential Reaching Law

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Abstract: As a typical multi-input and multi-output nonlinear system, spacecraft faces the challenges of rapid maneuvering at a large angle and can control rapidly as well as stably. However, there are several problems in the sliding mode control method commonly used in the aerospace field, such as slow convergence, singularity, and chattering. Taking consideration of aforementioned conditions, this paper proposes a Nonsingular Fast Terminal Sliding Mode Control based on Variable Exponential Reaching Law (NFTSMC-VERL). Firstly, using continuous hyperbolic tangent function instead of symbolic function, variable exponential reaching law is designed to solve the problem of chattering in exponential reaching law. The reachability and finite time convergence of this method are proved by theory. Secondly, a Nonsingular Fast Terminal Sliding mode (NFTSM) surface based on satellite attitude dynamics is designed as a sliding mode function. Thirdly, the controller is designed by combining the VERL with the NFTSM function. Finally, not only has the simulation results showed the good tracking performance of the spacecraft attitude control system, but also the chattering and singularity phenomenon of the traditional sliding mode controller is avoided by the designed controller. Besides, the robustness of designed controller is improved, compared with the traditional sliding mode controller.

Keywords: *Rapid attitude maneuver, Variable exponential reaching law, Fast convergence rate, Non-singularity*

1. Introduction

In recent years, with the continuous development of global space science and technology, the development of spacecraft has reached an unprecedented height. The rapid attitude maneuvering and stable control of payload as one of the key technologies of spacecraft are increasingly concerned by scholars[1]. However, the attitude dynamics of the spacecraft is nonlinear. Therefore the attitude control problem of the spacecraft becomes a nonlinear control problem[2], which has a serious impact on large-angle rapid maneuvering and rapid stability [3-4] .

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Sliding Mode Control (SMC) has been used by many scholars for precise spacecraft attitude control [5-8] because of its strong robustness. However, the traditional sliding mode control has problems such as infinite convergence of tracking error, so the Terminal Sliding Mode (TSM) which can converge in finite time was proposed and gained extensive attention[9]. On the basis of TSM, combining with reaching law, the quality of controller reaching motion and sliding mode motion can be improved as a whole, which has attracted the attention of scholars.

In reference[10], exponential reaching law was combined with Nonsingular Terminal Sliding Mode (NTSM) for the first time, which improved the convergence speed of the system and shortened the adjustment time on the basis of overcoming singular problems. However, as the sign function of exponential reaching law will cause chattering when approaching the sliding surface, it is particularly important to improve the exponential reaching law and weaken chattering. In reference[11], the improved saturation function was used to replace the sign function, which weakened the chattering and then combined with the sliding surface. In reference[12], based on the exponential reaching law, an adaptive variable speed exponential reaching law was designed, and then combined with the NTSM surface. In reference [13], the chattering was suppressed by changing the constant velocity reaching term into the variable speed reaching term, which was combined with the sliding surface.

However, in the design process of the controller mentioned above, the methods to solve exponential reaching law chattering were too complex, and the selection of sliding surface did not consider the fast convergence. Therefore, when designing the controller, this paper has proposed to use the continuous hyperbolic tangent function instead of the symbol function to suppress the chattering of exponential reaching law. Secondly, when choosing the sliding surface, the NFTSM surface is selected based on the satellite attitude described by the modified Rogers parameter (MRP) [14]. Then the controller is designed by combining the VERL with the NFTSM. Finally, under the same conditions, not only has the simulation results showed the good tracking performance of the spacecraft attitude control system, but also the chattering and singularity phenomenon of the traditional sliding mode controller is avoided by the designed controller.

This paper is organized as follows. Spacecraft attitude kinematics and dynamics is introduced in section 2. Variable Exponential Law is presented in section 3. A

NFTSM-VERL controller for attitude fast maneuver is designed in section 4. The simulation results are depicted in section 5. Finally, some conclusions are given in section 6.

2. Spacecraft Attitude Kinematics and Dynamics

The spacecraft attitude dynamics can be composed of kinematic equations and dynamic equations[15]. In the body coordinates, based on the MRP attitude description, the nonlinear attitude kinematics and dynamics equations of a rigid spacecraft external disturbances can be summarized as [16]

$$\dot{\sigma} = G(\sigma)\omega \tag{1}$$

$$\dot{\omega} = J^{-1}(-\omega^{\times} J\omega + u + d) \tag{2}$$

where, $J \in R^{3\times3}$ is the inertia matrix of spacecraft. $\sigma = [\sigma_1,\ \sigma_2,\ \sigma_3]^T \in R^3$ is the MRP vector, $\omega = [\omega_1,\ \omega_2,\ \omega_3]^T = [\omega_x,\ \omega_y,\ \omega_z]^T \in R^3$ is the angular velocity vector. $u = [u_1,\ u_2,\ u_3]^T = [u_x,\ u_y,\ u_z]^T \in R^3$ denotes the three-axis control torque of the spacecraft, which can be generated by the flywheel, thruster, magnetic torquer and other actuators of the satellite. $d = [d_1,\ d_2,\ d_3]^T = [d_x,\ d_y,\ d_z]^T \in R^3$ denotes bounded disturbance vector acting on rigid body spacecraft

The notation $\omega^{\times}$ is used to denote a skew-symmetric matrix generated by ω, having the following structure

$$\omega^{\times} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \tag{3}$$

The matrix function $G(\sigma)$ is given by

$$G(\sigma) = \frac{1}{4}\left[(1-\sigma^T\sigma)I + 2\sigma\sigma^T + 2\sigma^{\times}\right] \tag{4}$$

$I \in R^{3\times3}$ denotes a 3×3 dimensional identity matrix.

d is a bounded function satisfying

$$\| d \| \le \rho \tag{5}$$

where ρ denotes the upper bound of disturbances.

3. Variable Exponential Reaching Law Design and Analysis

3.1 Variable Exponential Reaching Law Design

The motion of sliding mode variable structure control consists of two stages : The first stage is that the moving point tends to switch from an arbitrary initial state to a switching surface, which we called this phase approaching motion. The second stage is the stage in which the moving point moves along the switching surface toward the stable point, which we called this phase sliding mode motion. Since there is no limit to the trajectory in the approaching process, the approach of the reaching law can be used to improve the dynamic quality of the approaching motion. In the 1990s, Academician Gao Wei-bing first proposed the concept of exponential reaching law [17]. Exponential reaching law is as follows:

$$\dot{s} = -ks - \varepsilon \operatorname{sgn}(s) \qquad k > 0, \varepsilon > 0 \tag{6}$$

where $\dot{s} = -ks$ is the exponential reaching term and $\dot{s} = -\varepsilon \operatorname{sgn}(s)$ is the constant velocity reaching term. Using the exponential reaching law can not only make the moving point reach the sliding surface quickly at the constant velocity and exponential speeds, but also make the speed of the moving point reach the sliding surface very small, and improve the dynamic quality of the system.

However, since the moving point reaches the sliding surface, the constant velocity approaching term $\dot{s} = -\varepsilon \operatorname{sgn}(s)$ is not equal to zero, so a high frequency buffeting band with a bandwidth 2ε is formed near the origin, which increases the burden on the controller. Therefore, the authors propose a method of using hyperbolic tangent function $\tanh(\mu \mathrm{s})$ to replace the sign function $\varepsilon \operatorname{sgn}(\mathrm{s})$. The variable exponential reaching law is as follows:

$$\dot{s} = -ks - \varepsilon \tanh(\mu s) \tag{7}$$

where $k > 0, \varepsilon > 0,$ μ is a positive constant. Since $\tanh(\mu \mathrm{s})$ is close to $\operatorname{sgn}(\mathrm{s})$ as μ increases, μ is limited to a range of intervals.

In terms of the variable exponential reaching law, the system motion point can reach the sliding surface quickly at both exponential and variable speeds, ensuring that the sliding mode motion moves along the designed sliding surface. At the same time, when the system motion point reaches the sliding surface, the exponential term reaching term $\dot{s} = -ks$ is close to 0, and the shifting approach term $\dot{s} = -\varepsilon \tanh(\mu s)$ is getting smaller, so that the bandwidth 2ε of the switching band is continuously reduced, and finally the system motion mode is stabilized at the origin. , thereby suppressing chattering caused by sliding mode variable structure control.

3.2 Reachability analysis

The reachability condition of the sliding mode is

$$s \cdot \dot{s} < 0 \tag{8}$$

Substituting (5) into (8), it follows that

$$\begin{aligned} s \cdot \dot{s} &= s(-ks - \varepsilon \tanh(\mu s)) \\ &= -ks^2 - \varepsilon s \tanh(\mu s) \\ &= -ks^2 - \varepsilon \left| s \tanh(\mu s) \right| < 0 \end{aligned} \tag{9}$$

Therefore, this VERL satisfies the reachability condition.

3.3 Finite time to reach the sliding surface analysis

The system is divided into two stages from the initial state to the sliding surface $s = 0$: the first stage is from the initial position to the reaching switching surface. Since s is large, the exponential reaching term $\dot{s} = -ks$ plays a leading role in this process, ignoring the influence of the second term, and calculating the time t_1 required for the first stage.

$$\dot{s} = -ks \tag{10}$$

Integrating (10), it follows that

$$s = s(0)e^{-kt} \tag{11}$$

Thus

$$t_1 = -\frac{1}{k} \ln \frac{s}{s(0)} \tag{12}$$

The second stage is from reaching switching surface to the switching surface. Since s is small, $\dot{s} = -\varepsilon \tanh(\mu s)$ plays a leading role in the process, ignoring the influence of the first item, and calculating the time t_2 needed for the second stage.

$$\dot{s} = -\varepsilon \tanh(\mu s) \tag{13}$$

Since the second stage s is small, if $\tanh(\mu s)$ is approximated to μs, then (13) becomes

$$\dot{s} = -\varepsilon\mu s \tag{14}$$

Integrating (14), it follows that

$$s = s(0)e^{-\varepsilon\mu t} \tag{15}$$

Thus

$$t_2 = -\frac{1}{\varepsilon\mu}\ln\frac{s}{s(0)} \tag{16}$$

Considering the time obtained in equations (12) and (16) was obtained by ignoring one of the expressions of the reaching law, the total time required to reach the switching surface from the initial position should be:

$$\begin{aligned} t < t_1 + t_2 &= -\frac{1}{k}\ln\frac{s}{s(0)} - \frac{1}{\varepsilon\mu}\ln\frac{s}{s(0)} \\ &= \ln\left(\frac{s}{s_0}\right)^{-\frac{1}{k}-\frac{1}{\varepsilon\mu}} \end{aligned} \tag{17}$$

Since k, ε, μ are fixed constants, the VERL can reach the sliding surface in a finite time.

4. NFTSMC-IRL for Attitude dynamics

4.1 Sliding mode surface function Design

In order to avoid the singular problem of the traditional sliding mode and improve the convergence speed of the sliding mode system. Liang Yang [18] proposed the NFTSM method, and proved its characteristics of fast convergence in a limited time and avoiding singularity problem. The structure of NFTSM is given as

$$s(t) = x + k_1' \operatorname{sgn}^{\alpha_1'} x + k_2' \operatorname{sgn}^{\alpha_2'} \dot{x} = 0 \tag{18}$$

in which $k_1' > 0$, $k_2' > 0$, $\alpha_1' > \alpha_2'$, and $1 < \alpha_2' < 2$.

Based on (18), The structure of NFTSM with MRP and angular velocity can be defined as

$$s = \sigma + \Lambda_1 \operatorname{sgn}^{\Gamma_1} \sigma + \Lambda_2 \operatorname{sgn}^{\Gamma_2} \dot{\sigma} \tag{19}$$

where s denotes 3-dimensional sliding surface, and the involved matrices are represented by

$$\Lambda_1 = \mathrm{diag}(\lambda_{11}, \lambda_{12}, \lambda_{13}) \quad \Gamma_1 = \mathrm{diag}(\gamma_{11}, \gamma_{12}, \gamma_{13})$$
$$\Lambda_2 = \mathrm{diag}(\lambda_{21}, \lambda_{22}, \lambda_{23}) \quad \Gamma_2 = \mathrm{diag}(\gamma_{21}, \gamma_{22}, \gamma_{23}) \tag{20}$$

with $\lambda_{1i} > 0,\ \lambda_{2i} > 0,\ \gamma_{1i} > \gamma_{2i},\ 1 < \gamma_{2i} < 2$, for every $i = 1, 2, 3$.

$\mathrm{sgn}^{\Gamma_1} \sigma$ is a vector defined as

$$\begin{aligned} \mathrm{sgn}^{\Gamma_1} \sigma &= [\mathrm{sgn}^{\gamma_{11}} \sigma_1,\ \mathrm{sgn}^{\gamma_{12}} \sigma_2,\ \mathrm{sgn}^{\gamma_{13}} \sigma_3]^T \\ &= [|\sigma_1|^{\gamma_{11}} \mathrm{sgn}\,\sigma_1,\ |\sigma_2|^{\gamma_{12}} \mathrm{sgn}\,\sigma_2,\ |\sigma_3|^{\gamma_{13}} \mathrm{sgn}\,\sigma_3]^T \end{aligned} \tag{21}$$

$\mathrm{sgn}^{\Gamma_2} \dot{\sigma}$ is a vector defined as

$$\begin{aligned} \mathrm{sgn}^{\Gamma_2} \dot{\sigma} &= [\mathrm{sgn}^{\gamma_{21}} \dot{\sigma}_1,\ \mathrm{sgn}^{\gamma_{22}} \dot{\sigma}_2,\ \mathrm{sgn}^{\gamma_{23}} \dot{\sigma}_3]^T \\ &= [|\dot{\sigma}_1|^{\gamma_{21}} \mathrm{sgn}\,\dot{\sigma}_1,\ |\dot{\sigma}_2|^{\gamma_{22}} \mathrm{sgn}\,\dot{\sigma}_2,\ |\dot{\sigma}_3|^{\gamma_{23}} \mathrm{sgn}\,\dot{\sigma}_3]^T \end{aligned} \tag{22}$$

The time-derivatives of $\mathrm{sgn}^{\Gamma_1} \boldsymbol{\sigma}$ and $\mathrm{sgn}^{\Gamma_2} \dot{\boldsymbol{\sigma}}$ are given by

$$\begin{aligned} \frac{d}{dt}(\mathrm{sgn}^{\Gamma_1} \sigma) &= \Gamma_1 diag(|\sigma|^{\Gamma_1 - I})\dot{\sigma} \\ \frac{d}{dt}(\mathrm{sgn}^{\Gamma_2} \dot{\sigma}) &= \Gamma_2 diag(|\dot{\sigma}|^{\Gamma_2 - I})\ddot{\sigma} \end{aligned} \tag{23}$$

4.2 Control law design

In the design of the control law, the system will follow the variable exponential reaching law to reach the non-singular fast terminal sliding surface.

From (19), its time derivative can be given by

$$\dot{s} = \dot{\sigma} + \Lambda_1 \Gamma_1 diag(|\sigma|^{\Gamma_1 - I})\dot{\sigma} + \Lambda_2 \Gamma_2 diag(|\dot{\sigma}|^{\Gamma_2 - I})\ddot{\sigma} \tag{24}$$

Jointing (7) and (24), it follows that

$$\dot{\sigma} + \Lambda_1 \Gamma_1 diag(|\sigma|^{\Gamma_1 - I})\dot{\sigma} + \Lambda_2 \Gamma_2 diag(|\dot{\sigma}|^{\Gamma_2 - I})\ddot{\sigma} = -ks - \varepsilon \tanh(\mu s) \tag{25}$$

From (1), its time derivative can be given by

$$\ddot{\sigma} = \frac{d}{dt}(G(\sigma)\omega) = \frac{1}{2}\left[(\sigma^T \omega)I + \sigma\omega^T - \omega\sigma^T - \omega^\times\right]\dot{\sigma} + G(\sigma)\dot{\omega} \tag{26}$$

In order to simplify (26) , Define:

$$H(\sigma, \omega) = \frac{\partial \dot{\sigma}}{\partial \sigma} = \frac{1}{2}\left[(\sigma^T \omega)I + \sigma\omega^T - \omega\sigma^T - \omega^\times\right] \tag{27}$$

Substituting (27) into (26), it follows that

$$\ddot{\sigma} = H(\sigma, \omega)\dot{\sigma} + G(\sigma)\dot{\omega} \tag{28}$$

Jointing (1),(2), (25) and (28), it follows that

$$\left[\mathrm{I} + \Lambda_1 \cdot \Gamma_1 \cdot diag(|\sigma|^{\Gamma_1 - \mathrm{I}}) + \Lambda_2 \cdot \Gamma_2 \cdot diag(|\dot{\sigma}|^{\Gamma_2 - \mathrm{I}}) \cdot H(\sigma, \omega) \right] \dot{\sigma} + \\ \Lambda_2 \cdot \Gamma_2 \cdot diag(|\dot{\sigma}|^{\Gamma_2 - \mathrm{I}}) \cdot G(\sigma) \cdot J^{-1} \cdot (-\omega^{\times} \cdot J \cdot \omega + d) + ks + \varepsilon \cdot \tanh(\mu s) \\ = -\Lambda_2 \cdot \Gamma_2 \cdot diag(|\dot{\sigma}|^{\Gamma_2 - \mathrm{I}}) \cdot G(\sigma) \cdot J^{-1} u \tag{29}$$

In order to simplify(29), Define:

$$M = \Lambda_2 \cdot \Gamma_2 \cdot diag(|\dot{\sigma}|^{\Gamma_2 - \mathrm{I}}) \tag{30}$$

Substituting (5) and (30) into (29), we could get the following control law

$$u = -JG^{-1}M^{-1} \left\{ \begin{array}{l} \left[\mathrm{I} + \Lambda_1 \cdot \Gamma_1 \cdot diag(|\sigma|^{\Gamma_1 - \mathrm{I}}) + M \cdot H(\sigma, \omega) \right] \dot{\sigma} + \\ M \cdot G(\sigma) \cdot J^{-1} \cdot (-\omega^{\times} J \omega + \rho \mathrm{I}) + ks + \varepsilon \cdot \tanh(\mu s) \end{array} \right\} \tag{31}$$

5. Numerical Simulation

In order to verify the effectiveness of the algorithm proposed in this paper, both the traditional sliding mode control method and the algorithm proposed in this paper are simulated under the same physical conditions. The specific physical parameters are as follows [16] :

The inertia matrix of a rigid spacecraft

$$J = \begin{bmatrix} 420 & 18 & -15 \\ 18 & 256 & -12 \\ -15 & -12 & 618 \end{bmatrix} (\mathrm{kg} \cdot \mathrm{m}^2) \tag{32}$$

The initial angular velocity and MRP orientation are respectively set to

$$\sigma(0) = [0.3\ 0.2\ -0.3]^T,\ \omega(0) = [0\ 0\ 0]^T \tag{33}$$

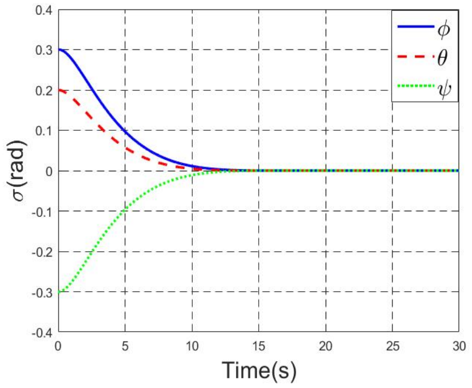
The external disturbance torque is assumed to be

$$d = \begin{bmatrix} 2 + 2\sin(0.042t) \\ 1 + 3\sin(0.042t) \\ 3 + 2\sin(0.042t) \end{bmatrix} \times 10^{-3} (N \cdot m) \tag{34}$$

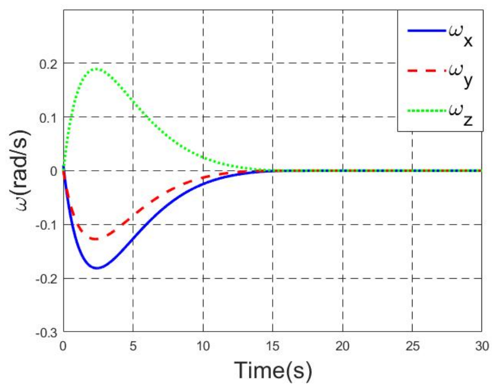
The coefficients in control law (31) are set to

$$\begin{array}{ll} \Lambda_1 = 0.9\mathrm{I} & \Lambda_2 = 8\mathrm{I} \\ \Gamma_1 = 0.7\mathrm{I} & \Gamma_2 = \mathrm{I} \\ K_1 = 0.8\mathrm{I} & \mu = 1000 \\ \rho = 0.005 & \varepsilon = 0.001 \end{array} \tag{35}$$

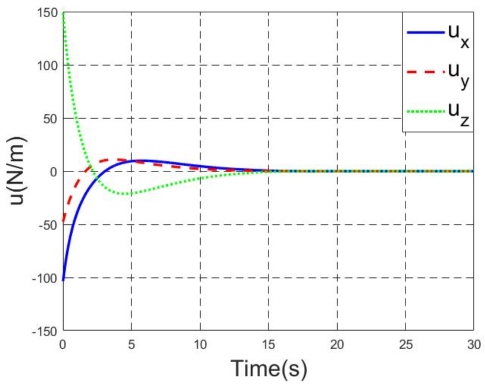
The simulation results of the proposed method are shown in Figure 1.



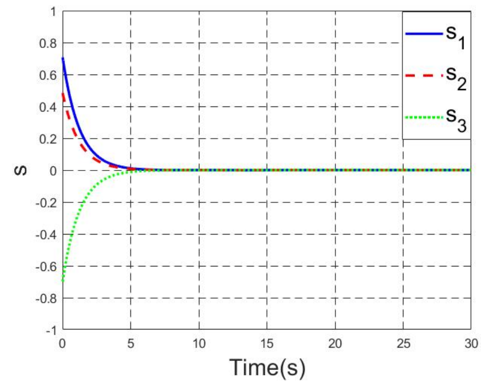
(a)Time history of MRP vector



(b)Time history of angular velocity



(c) Time history of control input



(d) Time history of sliding surface

Figure 1. Time history of spacecraft attitude

By analyzing Figure 1, we can get: The algorithm proposed in this paper solves the problem of chattering. In addition, under the same external conditions, the convergence time of the proposed algorithm is much less than that proposed in the reference [15], which indicates that the proposed algorithm in this paper has better fast convergence.

6. Conclusion

In view of the challenges of spacecraft rapid maneuver and rapid stable control in space flight, combined with the idea of reaching law, this paper proposes a design method of NFTSMC-VERL. On the basis of overcoming the chattering and singularity problems, the speed of approaching the sliding surface is improved, and the adjustment time is shortened. The simulation results show that the designed algorithm has the characteristics of fast convergence and strong anti-interference ability, which is more suitable for spacecraft attitude control.

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