Recent research progress on fluid-thermal-structural coupling analysis in hypersonic flows

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Abstract

The design of lightweight structures and thermal protection systems for hypersonic vehicles depends largely on the accurate and reliable prediction of the aerothermal loads, the structural temperatures and their gradients, and the structural deformations and stresses. Under the severe aerothermodynamic environment, there is a physical fact that significant interactions which is often referred as multi-physics coupling problems, could occur between the external aerodynamic flow field and the internal temperature field inside the structure through a fluid-solid interface. For practical applications, the authors conducted numerical modeling and analysis on some key issues (e.g. non-ablative thermal protection and aerothermoelastic problems) involved with hypersonic fluid dynamics, thermal dynamics and structural dynamics.

In the present paper, a framework of HyCCD (Hypersonic Coupling Computational Dynamics) was developed to integrate an independently developed program solving hypersonic aerthermodynamic simulation with a finite element analysis professional software. In consideration of the mathematical and physical description of multi-physics coupling mechanism, an adaptive coupling strategies was also proposed. Some representative coupling problem encountered in hypersonic flows were systematically analyzed to study the intrinsic fluid-thermal-structural coupling characteristics and mechanisms. The results can theoretically and technically support the studies on comprehensive performance assessment and optimization of thermal protection system and static or dynamic aerothermoelastic problem of hypersonic vehicles.

Keywords: Hypersonic flow, Multi-physics coupling, Aerothermodynamics, Aerothermoelasticity

1. Introduction

The hypersonic air-breathing vehicles have experienced a long-range maneuverable flight in the near- space atmosphere with a wide range of Mach numbers. It makes the aerothermodynamic environment extremely complicated, featuring complex flow field, high enthalpy, long duration and low/medium heat flux. In addition, the interaction between the aerodynamic force/aeroheating flux of the external flow field and the heat transfer/thermal stress/deformation of the physical field inside the thermal protection system (TPS) will become extremely strong. Furthermore, a large number of applications of lightweight flexible materials and large thin-walled structures, especially flight control rudder and other components, will lead to another aerothermoelasticity [1], and the influence of sustained aeroheating should be considered. Therefore, the coupling between multi-physics such as flow field, heat and structure should be considered for a new-generation of air-breathing hypersonic vehicle with the ability of hypersonic long-range maneuverable flight in the near-space atmosphere.

In this paper, the coupling characteristics and mechanism of multi-physics coupling problems such as fluid-thermal-structure coupling of hypersonic vehicle are studied and analysed systematically, a reasonable multi-physics coupling model is established, and effective coupling analysis strategies based on computational fluid dynamics (CFD) and computational thermaldynamics (CTD) and computational structural dynamics (CSD) are proposed, which provide theoretical support and analytical tools for the further study of key issues such as non-ablative thermal protection and aerothermoelasticity.

In order to improve the computational feasibility of high-fidelity modeling techniques and minimize the computational cost, the authors developed a time-marching simulation framework of hypersonic

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computational coupled dynamics (HyCCD). It had embedded with the time-adaptive, loosely coupled strategy, which proved the reliability and effectiveness of this strategy in the modeling and analysis of several representative multi-physics coupling problems such as inlet and wing of hypersonic vehicles [2-4].

2. The description of multi-physics coupling problem

As shown in Figure 1, the multi-physical coupling problem mainly involves fluid and solid, where complex physical processes occur between aerothermodynamics in fluid and thermo-structural dynamics within the solid through a fluid-solid coupling interface. The structural deformation of some structures with low rigidity cannot be ignored, which causes thermal stress and thermal deformation, while the coupling problem of fluid-thermal-structural is mainly manifested as the coupling of aerodynamic force/heat and heat transfer/thermal stress/deformation. Especially for large thin-walled flexible structures, such as wings and flight control rudders, aerothermoelastic problems are more prominent. In this structure, the inertial effect and vibration of the structure should be considered for fluid-thermal-structural coupling.

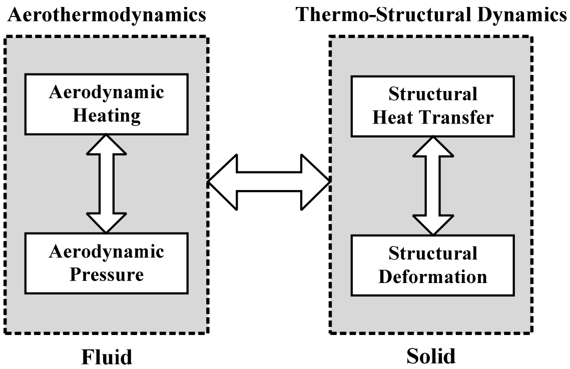


Figure 1. ***Coupling mechanism of multi-physics coupling problems for hypersonic vehicle.***

The so-called multi-physics coupling problem modeling mainly refers to the construction of mathematical-physical model to describe the coupling behavior of the multi-physical fields, that is, Partial Differential Equation systems (PDEs) to describe the multi-physics coupling problem and the corresponding initial/boundary conditions. And then, the analysis is to solve partial differential equations by numerical simulation to obtain physical properties and behaviors. According to the characteristics of multi-physics coupling problems, the global strategy for modeling and analysis is shown in Figure 2. The integral coupling method is used for aerodynamic force/heat coupling in fluid and thermo-structural dynamics within the solid. On the contrary, the separation coupling method is applied to the fluid-thermal coupling and fluid-thermal-structural coupling problems through the fluid-solid coupling interface.

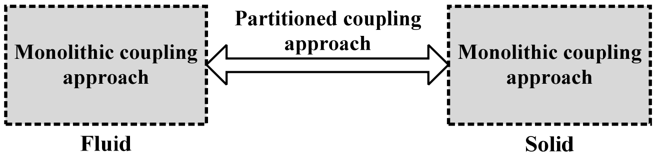


Figure 2. ***Global strategy for modeling and analysis approaches.***

3. Coupling analysis strategies based on dynamic flight trajectory

The coupling mechanism of fluid-thermal coupling problem (or fluid-solid conjugate heat transfer problem) is a physical process of interaction between aeroheating in fluid and heat transfer within the solid through fluid-solid coupling interface. The aeroheating environment under thermal load is an active excitation, which changes continuously along the flight trajectory, making the problem of fluid-thermal coupling a sustained non-transient physical coupling process.

This coupling process involves three different time scales, namely, the characteristic time of dynamic flight trajectory, the characteristic time of flow response and the characteristic time of structural thermal response. The research and development of coupling analysis strategy should fully consider the above obvious differences in the above time scales. There are two concepts herein: 1) static flight trajectory, that is, the flight state (altitude, speed and Angle of attack) remains constant with time; 2) dynamic flight trajectory, that is, flight state (altitude, speed and Angle of attack) changes continuously with time.

For fluid-thermal-structural coupling and structural thermal modals, considering the continuously variation with time of the flight trajectory of the vehicle, the influence of the flight trajectory variation should be considered in the coupling analysis. Therefore, the authors only discuss coupling analysis strategy based on dynamic flight trajectory in this paper.

Since the variation of flight trajectory is the macroscopic motion of the vehicle as a rigid body, and the time span of continuous flight is large, the variation of flight trajectory with time is relatively slow. Therefore, the time scale that affects the aeroheating characteristics is much larger than that of the coupling of flow field and heat transfer. The coupling analysis strategy of hypersonic fluid-thermal coupling problem based on dynamic flight trajectory is shown in Figure 3.

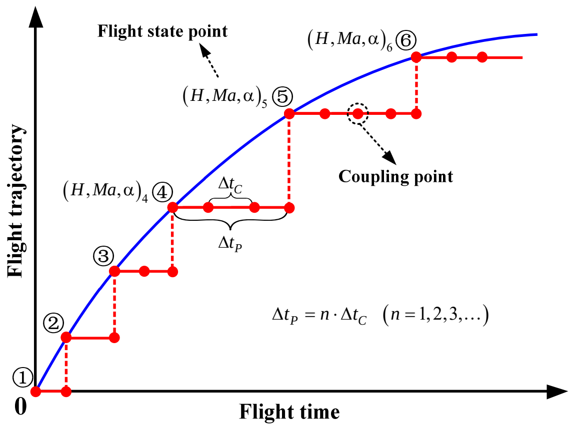


Figure 3. ***Coupling analysis strategy based on dynamic trajectory.***

The fluid-thermal coupling problem of the sustained dynamic flight trajectory is discretized into a set of fluid-thermal coupling problems of quasi-static flight trajectories in chronological order. The details are listed below:

1) The flight trajectory of the vehicle is regarded as a continuous flight state function in the time domain. The appropriate time step Δt_P is selected to discretize the flight trajectory into a series of discrete flight state points $(H, Ma, \alpha)_i$ and the duration of each discrete flight state point is the internal flight state which remains constant, and is set as the average of flight state at the starting point and at the ending point of the discrete flight state.

2) During Δt_P of each discrete flight state, the problem is regarded as the fluid-thermal coupling problem based on a static flight trajectory. The appropriate coupling calculation time step $\Delta t_C = \Delta t_P/n \ (n = 1,2,3 \cdots)$ is selected.

3) after the coupling calculation of one discrete flight state point, the coupling calculation of the next discrete flight state point will be conducted in time sequence until the end of the entire flight.

3.1. Modeling and analysis of fluid-thermal-structural coupling problem

The modeling and analysis of fluid-thermal-structural coupling problem can be generally divided into monolithic coupling approach and the partitioned coupling approach[5,6]. According to the coupling characteristics, the fluid-thermal-structural coupling model is shown in Figure 4. It represents a strong bi-directional coupling relationship between the aerothermodynamic environment of external flowfield and the structural thermal response of internal solid structures[2].

A unified fluid governing equation is adopted to describe the volumetric coupling between aerodynamic forces and aerodynamic heat in the fluid flow field, and aerodynamic forces/heat parameters are obtained through computational fluid dynamics (CFD). The thermal load (wall heat flux) and force load (wall pressure p) are applied on the solid through the fluid-solid coupling interface. Within the solid, the thermal response is described by the heat conduction governing equations, while the stress/strain is described by the thermoelastic governing equations. Considering the effect of temperature-deformation coupling, the internal parameters of solid can be obtained by solving the heat conduction and thermoelastic governing equations with HyCCD platform.

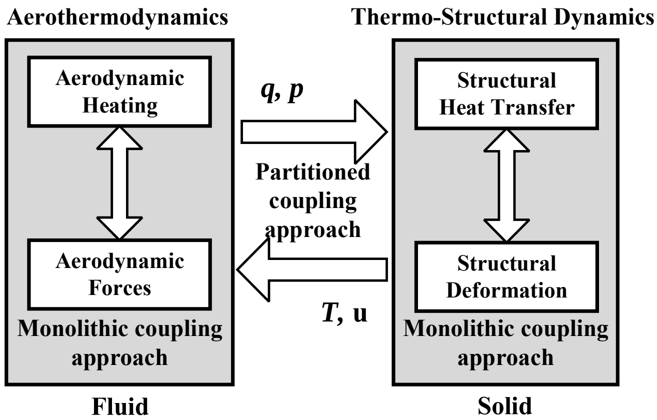


Figure 4. ***Fluid-thermal-structural coupling model.***

Through the fluid-solid coupling interface, the temperature condition (the wall temperature) and structural deformation condition (the surface displacement $\mathbf{u}$) are provided for the fluid.

The flow response characteristic time can be expressed as follows:

$$\tau_F = \frac{L}{U_\infty} \tag{1}$$

where U_∞ is the freestream velocity, L is the characteristic length scale of hypersonic vehicles from local structural deviation to the overall length. The structural thermal response characteristic time can be expressed as:

$$\tau_T = \frac{\delta^2}{\alpha_S} \tag{2}$$

where $\alpha_S = \lambda_S/(\rho_S C_{pS})$ is the thermal diffusivity of solid structure, δ is the characteristic length scale of solid structure such as its thickness. For hypersonic fluid-thermal-structural coupling problems, the characteristic time of structural thermal response is much longer than that of flow response. Therefore, when hypersonic flow is disturbed, it can reach a steady state instantaneously without relaxation process. Based on this physical assumption, the partitioned coupling method for calculating steady flowfield and transient structural heat transfer respectively can greatly improve the computational efficiency of coupling analysis.

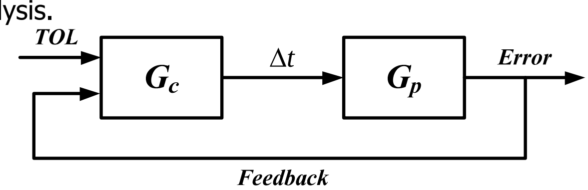


Figure 5. ***Stepsize control strategy viewed as an automatic control problem.***

The fluid-thermal-structural coupling strength between the flowfield and structures will be gradually weakenen and eventually reach the equilibrium state as time goes on. According to the physical time evolution of the coupling problem, it is necessary to select an appropriate coupling time stepsize Δt_c to improve the accuracy and efficiency of long-duration coupling analysis. Gustafsson et al. [7-9] proposed a time stepsize control algorithm for the time-marching solution of the ordinary differential equations, taking the time step selection as a standard automatic control problem. As shown in Figure 5, the plant G_p consists of the differential equations and their time integral program, taking a stepsize Δt as the input and generating an estimated error $Error$ as the output. G_c represents the time step controller, which attempts to select a suitable stepsize so that the local estimated error at each step is as close as possible to the prescribed tolerance TOL.

In this paper, the control algorithm of coupling time step in fluid-thermal-structural coupling is extended. Therefore, selecting coupling time stepsize Δt_c is considered an automatic control problem. By using the proportional-integral-derivative (PID) controller, it can be defined as:

$$\Delta t_c^{n+1} = \left(\frac{r_{n-1}}{r_n}\right)^{K_P} \left(\frac{TOL}{r_n}\right)^{K_I} \left(\frac{r_n^2}{r_n r_{n-2}}\right)^{K_D} \Delta t_c^n \tag{3}$$

where K_P, K_I, and K_D are the empirical parameters, which can equal to 0.075, 0.175 and 0.01 recommended in [10] respectively, and r_n is the change in concerned physical quantities at time t_c^n, which can be defined by considering the temperature change at the coupling interface:

$$r_n = \left(\frac{\int_{\Gamma_{FS}} (T_n - T_{n-1})^2 d\Gamma}{\int_{\Gamma_{FS}} T_n^2 d\Gamma} \right)^{1/2} \tag{4}$$

Furthermore, for an explicitly integrated problem, the stepsize will be limited by the numerical stability requirement. To prevent the excessive growth or reduction of the coupling time stepsize, a stepsize limiter is introduced as follows:

$$\Delta t_{min} \leq \Delta t_c^{n+1} \leq \Delta t_{max} \tag{5a}$$

$$m \leq \frac{\Delta t_c^{n+1}}{\Delta t_c^n} \leq M \tag{5b}$$

Finally, the loosely-coupled analysis strategy embedded with the adaptive coupling time stepsize approach can be described as illustrated in Figure 6. The detailed implementation procedures are summarized as follows:

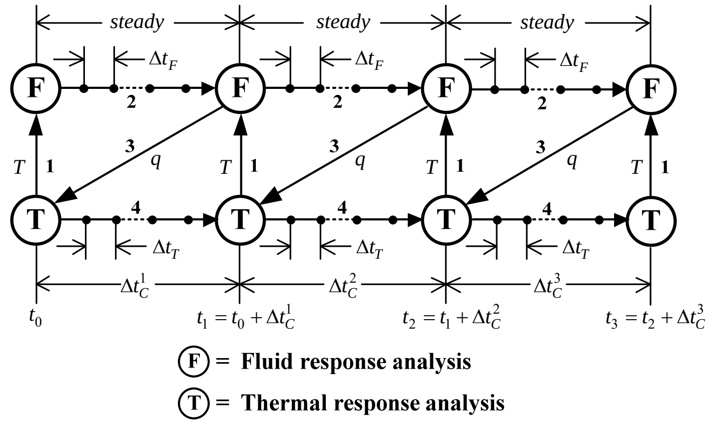


Figure 6. ***Loosely coupled analysis strategy embedded with the adaptive coupling time stepsize.***

(1) For the initial coupling step $n = 1$, the input tolerance TOL and the parameters of the stepsize limiter Δt_{min}, Δt_{max}, m, and M need to be prescribed according to the accuracy and efficiency requirements respectively, and an initial coupling time stepsize Δt_c^1 also needs to be given initially;

(2) For the coupling step $n = 2$, r_1 is calculated by equation (4) according to the interface temperature of the initial coupling step $n = 1$, , and then the coupling time Δt_c^2 is calculated by equations (3), (5) and (6). The PID controller parameters are $K_P = 0$, $K_I = 0.175$, and $K_D = 0$.

(3) For the coupling step $n = 3$, r_1 and r_2 are calculated by equation (4) according to the interface temperature of the initial coupling step $n = 2$, and then the coupling time Δt_c^3 is calculated by equations (3), (5) and (6). The PID controller parameters are $K_P = 0.075$, $K_I = 0.175$, and $K_D = 0$.

(4) Similarly, for the coupling step $n \geq 4$, r_{n-2}, r_{n-1}, and r_n are calculated by equation (4) according to the interface temperature of the initial coupling step $n - 1$, and then the coupling time is calculated by equations (3), (5) and (6). The PID controller parameters are $K_P = 0.075$, $K_I = 0.175$, and $K_D = 0.01$.

The coupling analysis strategy based on dynamic trajectory is to discretize the sustained dynamic trajectory into a finite number of quasi-static trajectories in time sequence.

3.2. Thermal modal analysis strategy based on multi-physics coupling

The thermal modal analysis strategy based on multi-physics coupling integral method is shown in Figure 7. It can be summarized as follows: 1) The transient temperature field and stress field within the solid structure along its static or dynamic trajectory are obtained by using the multi-physics coupling integral method based on CFD, CTD and CSD through the hypersonic fluid-thermal-structural coupling analysis according to the above coupling analysis strategy. 2) thermal rigidity matrix can be constructed by coupling calculation of thermodynamic state parameters within the solid structure at each time point. Then the generalized eigenvalue problem is solved by the method of constant scale state analysis.

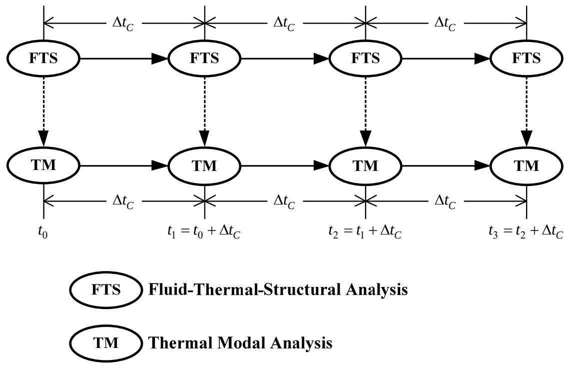


Figure 7. ***The thermal modal analysis strategy based on multi-physics coupling integration.***

4. Results and discussions

4.1. Fluid-thermal-structural coupling analysis of inlet cowl leading edge

As for the fuselage-engine-integrated design, in order to generate lift and obtain the flow required by the engine inlet, the compressor precursor is utilized for precompression. In this case, both the waverider forebody and the cowl leading edge are on windward side where the most severe aeroheating takes place[2,4]. As shown in Figure 8, the oblique shock waves of precursor preloading and the shock waves at the cowl leading edge may cross each other, leading to shock interaction, which may aggravate the aeroheating near the cowl leading edge, even more seriously than the nose leading edge. Case 1 is defined as the over-ideal state, that is, the incident shock enters the cowl and the cowl leading edge is under the far-field freestream state. Case 2 is defined as an ideal state, in which the incident shock just arrives the inlet cowl and interacts with the shock at the cowl leading edge. Case 3 is defined as under-ideal state, in which the outer incident shock hasn't reached the cowl and the cowl leading edge is under the downstream shock freestream state.

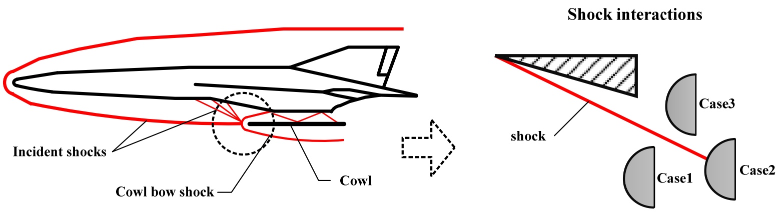


Figure 8. ***The shock interaction phenomena near the inlet cowl leading edge.***

The cylindrical leading edge model is used as the inlet cowl leading edge model, which use Titanium alloy (Ti-6Al-2Sn-4Zr-2Mo) as the material. Sustained coupling calculation time of 11 seconds is selected for the fluid-thermal-structural coupling calculation and analysis of the engine cowl leading edge model; the loosely-coupled analysis strategy is employed for calculation and its fluid-solid surficial coupling calculation time step adopts the adaptive strategy. The external flow field is calculated by high temperature chemical non-equilibrium gas model, non-catalytic wall is selected as the wall catalytic condition. The initial temperature for calculation of thermal-structural coupling within the solid domain is 300K with zero initial stress and the reference temperature of thermal stress is 300K. The aerodynamic force/thermal load of the external flowfield are considered and the wall radiation effect is also considered with the surface emissivity ε =1.0 in the coupling calculation and analysis. A pressure load p_∞ =1197 Pa is imposed on the inner wall, the fixed support is selected at both ends of the model.

It can be intuitively seen from Figure 9 that, under the effect of the striking of the extremely densified heat flux generated by shock interaction, the heat rapidly accumulates within the structure nearby the struck point, resulting in a leading increase in the temperature of the point. As time goes on, the heat accumulates at the point, the structure temperature is also increasing and the heat is gradually transferred to internal area in depth at the same time. Hence, the temperature distribution is also gradually expanding from the struck point to internal structure area in depth. Simultaneously, if the wall radiation effect is taken into account, the amplitude of overall temperature distribution within the structure declines greatly, because it effectively limits the heat entering the internal structure. Figure 10 shows that the earliest stress concentration occurs within the structure near the struck point on the wall. As time goes on, the stress distribution gradually extends to the internal structure area. At the same time, it is shown that the amplitude of overall stress distribution within the structure declines with the consideration of wall radiation effect.

Generally speaking, the aerodynamic force and thermal load have a great impact on the inlet cowl leading edge. Although the time applied in actual flight is very short, it is enough to cause thermal and dynamic damage to the thermal protection structure, posing severe challenges to material selection and structural design of thermal protection. As for the air-breathing hypersonic vehicles, the impact of shock interaction is very common in the surrounding flowfield. Therefore, in addition to the nose and the engine cowl leading edge, thermal protection design of local leading edges (structures such as the tail and rudders) on the windward side wrapped by the nose shocks should be carried out carefully.

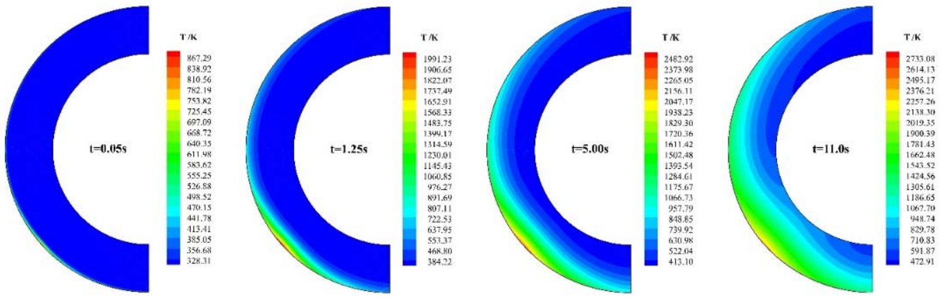


Figure 9. ***The structure temperature within the inlet cowl leading edge model.***

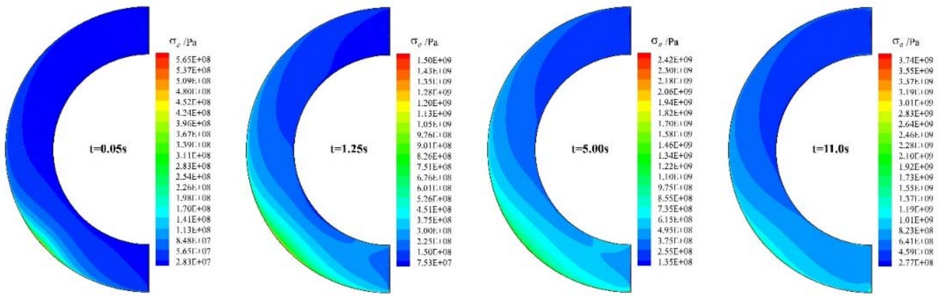
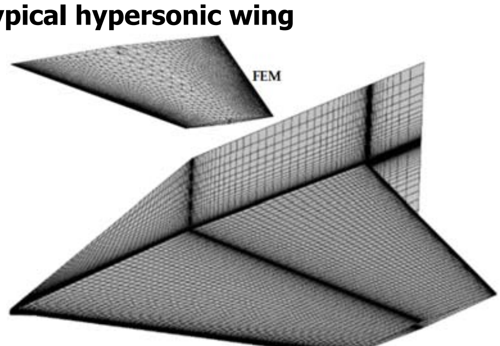


Figure 10. ***The structural stress within the inlet cowl leading edge model.***

Aeroelastic problems have always been the key to vehicle design. With the development of vehicles aeroelastic problem has gradually become a significant obstacle to the improvement of vehicle performance. Under the effect of aeroheating, the increase of the temperature of a structure leads to variation in physical parameters of its material. In addition, nonuniform temperature field within the structure causes prominent temperature gradients, which produces subsidiary thermal deformation and thermal stress, greatly alters the structural rigidity, and thus changes the natural vibration performance of the structure. The variation of natural vibration performance due to thermal load significantly affects vehicle pitch, flutter and control characteristics, and these effects are often unfavourable.

4.2. Thermal modal characteristic analysis of a typical hypersonic wing



(a) Hypersonic wing model (b) Grids for CFD and FEM

Figure 11. ***The three-dimensional low-aspect-ratio hypersonic wing and computational grids.***

The thermal modal characteristics analysis of a typical three-dimensional hypersonic wing model with low aspect-ratio under sustained flight is shown in Figure 11(a)[3,4]. It is a symmetrical double edge with the leading edge blunted and the trailing edge kept small thickness to avoid sharp edges. The mass density of the material is 4539 kg/m^3, the Poisson number is 0.32, and all other physical properties vary with temperatures. The initial temperature is 300K, the initial stress is zero, and the reference temperature of thermal stress is 300K. Fixed support is adopted at the wing root.

For the coupling modeling and analysis, the CFD computational grid around the wing is a multi-block structured grid with about 3×106 cells, as shown in Figure 11(b). The grid has 181×101 points on the wing surface, 101 points extending along the wingspan from the root, and 101 points extending radially outward from the surface. The majority of grid nodes are clustered around the wing surface and boundary layers, which can satisfy with the grid convergence. Here, for clarity, it only displays the symmetric plane at the wing root and the wing surface with one-point skip in three-directions. The FEM computational grid in solid is an unstructured grid in Figure 11(b), which has about 3×106 cells with the minimum grid element edge length 0.5mm. The two grids do not match with each other at the fluid-solid interface.

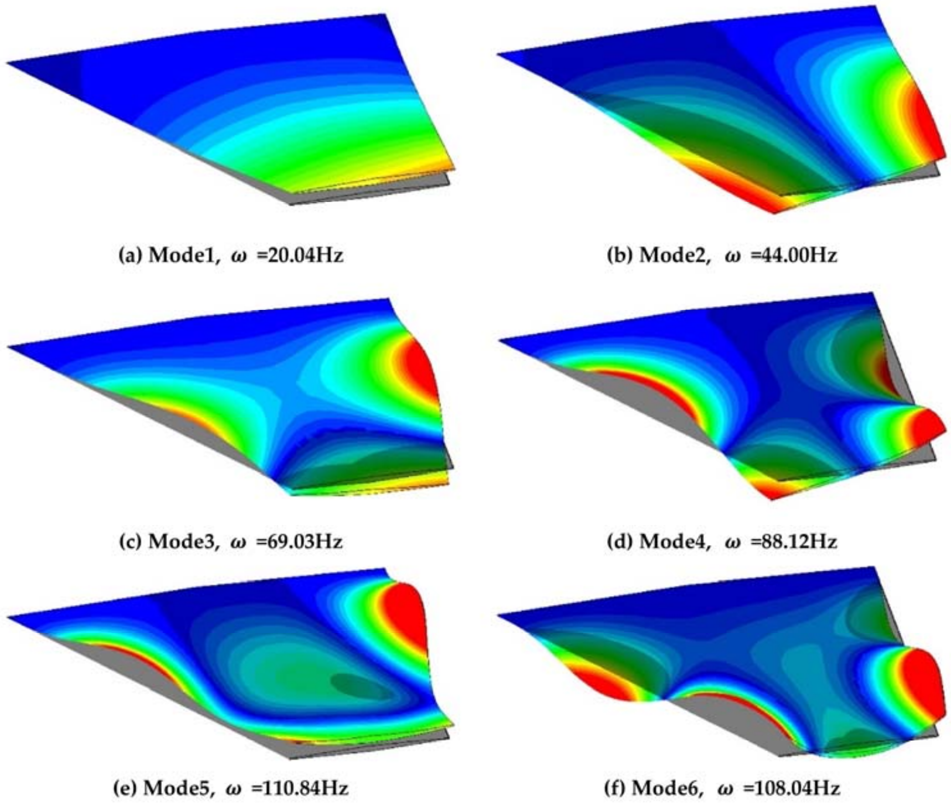


Figure 12. ***The first six vibration modes and frequencies of the hypersonic wing at initial time.***

Figure 12 shows the first six vibration modes and frequencies of the hypersonic wing at the initial time, indicating that the first mode is pure bending mode and the second mode is pure torsion mode.

A simple flight trajectory is assumed to further analyze variation of thermal modal characteristics in complicated flight. Figure 13 presents that dynamic variation of flight trajectory leads to the variation of the thermodynamic state within the wing as well as its modal frequency. From the initial time to the end of cruise, the modal frequency of each order declines gradually; the decline tends to be mild and then shows a recovery as the vehicle descends. It turns out that the first modals changes little along the flight trajectory, usually retaining the original modal shapes, while the major variation takes place in the 5th/6th-order mode, especially the 6th-order mode, in which case the bending-torsion coupling tends to occur. If the climbing, cruising and descending phase, especially cruising, last long enough, the modal frequencies and modal shapes of each order can have more significant variation with dynamic variation of the flight trajectory.

The modal frequencies of each order decreases with time, and the decreasing rates vary from mode to mode. Modes of all orders are in varying degrees close to or farther apart from each other over time, which may have a serious impact on the natural vibration characteristics of the structure. The results indicate that sustained aeroheating has effect more easily on modal shapes of higher order. Therefore, for hypersonic vehicles with large thin-walled control surfaces, sustained aeroheating has great effect on the natural vibration characteristics. Generally speaking, the strategies and methods developed for multi-physics field coupling integration analysis can effectively predict and analyze the variation of natural vibration characteristics (natural frequency and natural vibration shape), which lays a good foundation for further study on aerothermoelastic problems.

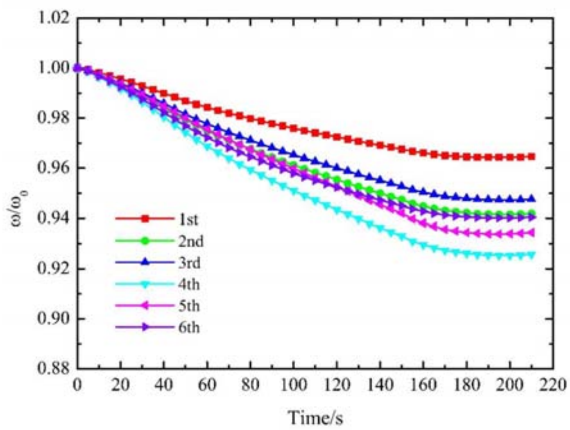


Figure 13. ***The first six modal frequencies along the flight trajectory***

5. Conclusions

Through the modeling and analysis of hypersonic multi-physics coupling problems, the mathematical and physical descriptions of various coupling models are established, and corresponding coupling analysis strategy is proposed. In the framework of coupling analysis strategy, hypersonic numerical simulation program and general finite element thermal analysis software are used to develop an integrated analysis program platform HyCCD, and the problems of hypersonic multi-physics coupling, coupled heat transfer and thermal-structural behavior of leding edges are studied, and finally the thermal modal characteristics of a representative low-aspect-ratio hypersonic wing along flight trajectory is investigated. Through these representative studies, the following key novel contributions have been obtained: 1) the coupling mechanism of the multi-physics coupling problems among hypersonic flow, thermal and structure is studied, and the multi-physics coupling mathematical models are established hierarchically; 2) an effective coupling analysis strategy is proposed, and a set of high-effective multi-physics coupling integrated analysis method that has engineering applicability are developed synthetically. These achievements can provide theoretical and technical support for the studies on comprehensive performance evaluation and optimization of thermal protection system and the study on aerothermoelastic problem.

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